Cappella dei Principi in Firenze, Italy: Experimental Analyses and Numerical Modeling for the Investigation of a Local Failure

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Abstract: The paper reports the results of a research aimed at analyzing and interpreting the cracking pattern on the Cappella dei Principi (Prince’s Chapel, the Medici’s mausoleum) in the Basilica of San Lorenzo (Florence, Italy). The research was motivated by the sudden collapse of a keystone of an internal barrel vault sustaining one of the lateral apses. After a brief description of the geometry of the structure, the principal results obtained from in situ surveys (flat-jack tests and cored samples) are described; then the numerical analyses developed both to obtain the static identification of the monument and to assess the possible causes leading to the observed failure are illustrated. The numerical modeling operation has been performed step by step, from linear and quite simple models built with plane elements up to a nonlinear model with three-dimensional elements. The nonlinear FEM, which has been tuned by utilizing the results of the in situ measures, is allowed to both qualitatively and quantitatively reproduce the behavior of the structure and its static problems in the area of the barrel vault over the side apses, allowing for justification of the manifested damage. The comprehension of the structural behavior allows identification of a proper retrofitting strategy.

CE Database subject headings: Masonry; Domes (structural element); Nondestructive tests; Finite element method; Structural failures; Italy; Numerical models.

Author keywords: Historical masonry dome; In situ nondestructive tests; Numerical modeling; FEM identification; Structural identification; Failure assessment.

Introduction

The Cappella dei Principi, the Medici’s mausoleum (a funeral chapel for members of the Medici family) near the Basilica of San Lorenzo in Florence (Italy), is one of the major representative monumental buildings in Florence both for its dimensional magnificence and its marble covering opulence (Ippolito 1998). The Cappella (chapel) was originally conceived by the Grand Duke Ferdinando I Medici’s (1549–1609) as a family mausoleum, and the work began in 1604 under the supervision of the architect Matteo Nigetti (~1560–1648); a disciple of Bernardo Buontalenti). The entire structure (crypt, chapel, tambour, and dome) was built over about 40 years (the work finished ~1648), and for importance and size, the octagonal masonry dome surmounting the chapel is the second one in Florence after the Dome of Santa Maria del Fiore (Gurrieri 1995).

The construction steps are quite clear. A marble plaque placed inside the chapel reports the date of January 10, 1604, as the construction start date. In 1608, the foundation and the edification of the crypt below the chapel were completed, and after 3 years, in 1611, most of the lateral walls of the octagonal chapel were finished. In parallel with the structural works for the edification of the chapel, in 1613, the internal marble covering was completed in the lowest part of the chapel (granite and porphyry with inlay of mother-of-pearl and lapis lazuli) made by the Opificio delle Pietre Dure, an institution still active specifically created by the Grand Duke Ferdinando I for the realization of this opera. The construction of the dome was initiated in 1625 and probably finished before the death of the architect Nigetti, because in 1654 historical documents report the payment for the removal of the wooden formwork used during the construction of the Dome. The shape of the mausoleum, as it was in 1654 [Fig. 1(a)], remained untouched until 1740 when Anna Maria Luisa de’ Medici, the Eletrice Palatina, commissioned the architects Ferdinando and Giuseppe Ruggieri to modify and renovate the external surface of the chapel according to the architectonical style of the time. In particular, the circular eyes of the tambour were turned into bell-shaped windows [Fig. 1(b)] by means of a partial and local demolition of the tambour, and a new (third) external thick dome was added to cover and protect the existing one against the environmental loads. At the same time, other openings existing at the basement of the tambour were closed. In 1743, the death of Anna Maria Luisa, the last descendant of the Medici’s family, stopped the work, and therefore the lantern and the marble ribs at the corner of the web of the octahedral dome were never built. Even if, from a structural point of view, the year 1743 can be assumed as the conclusive date for the construction of the chapel, the works had been continuing until the 1960s (~1962), when the covering marble of the chapel floor, which began in 1882, was completed with the laying of the last stone. Interested readers can refer to Baldini and Nardini (1984) and Bertani (1998) for more details.

The situation remains almost unchanged until November 4, 1999, when a marble slab of the internal covering in one of the lateral
apses dropped down, revealing the presence of static problems on the construction, because one of the keystones of the barrel vault over the lateral apses supporting the tambour had fallen. As a consequence of this local failure, the Soprintendenza ai Beni Ambientali e Architettonici di Firenze, Prato e Pistoia (Ministry Institution for Environmental and Architectural Heritage of Florence, Prato and Pistoia, the institution asked to preserve the cultural heritage in the Florence metropolitan area, referred to as Soprintendenza subsequently) started and coordinated a research program to evaluate the static health condition of the monument and to freeze/repair the damage. Within this overall activity, the Department of Civil and Environmental Engineering of the University of Florence was asked to contribute to the conservation of the monument by analyzing the static safety condition of the chapel with a specific research program.

According to this program, and in parallel with the removal of the internal marble covering (whose thickness is about 20–25 cm) that hid the structural masonry texture in the area of the damaged apse, a series of in situ investigations was started and a monitoring system was installed on the chapel after January 2002, in order to get a complete description of the building behavior.

The paper presents a selection of the main results of this research program, focusing the attention on the keystone failure. After an overview of the structural configuration and a discussion of the main geometrical and architectonical characteristics of the monument, the paper reports a description of the damage that was visible after the removal of the marbles covering the lateral apse. The results of the static in situ investigation (flat-jack tests in the apse area) are discussed in succession with the structural analyses for the local failure interpretation. The numerical modeling proceeded by steps, starting from simple linear models (developed with plane elements) up to nonlinear three-dimensional (3D) models (developed with solid and gap elements), taking into account the increased level of knowledge acquired during the experimental survey. As a matter of fact, the experimental investigation of the monumental building consisted of an iterative procedure where the typology and the extension of experimental tests have been combined with the results of preliminary numerical models of the building itself. The preliminary elastic analyses of the entire monument, even though this simplification can be considered to a large extent unrealistic, were planned to analyze the load-transfer pattern activated in the chapel by assuming variable stiffness ratios between the main structural elements (dome, tambour, lateral walls). The subsequent nonlinear numerical model, developed on the basis of a more realistic masonry mechanical behavior and by using the results of the investigation, was employed to refine the identified structural behavior, and it allowed the assessment of the principal schemes of transfer of the static loads. The main goal of this research program was to provide reliable interpretations of the actual damage (a diagnosis) to propose possible proper strengthening (i.e., respectful of the cultural value of the fabbrica [ICOMOS 2001] and effective with respect to the causes of the localized failure). The final nonlinear 3D numerical model, being able to reproduce the building behavior both qualitatively and quantitatively, once integrated with the results of the monitoring, will allow assessment of the behavior of the structural compound under possible severe loads, such as seismic actions.

Substantially, through the discussion of an illustrative case study, the paper aims at emphasizing both the role of numerical modeling and engineer judgment in the analysis and interpretation of structural failures. The importance (both for researcher and practitioner) of analyzing actual case studies in this field has been recently pointed out by Adam and Pallares (2010), who presented a special issue of engineering structures devoted to the study of actual structural failures. The purpose and the importance of “failure literacy” have been also discussed by Delatte (2010), who proposed its implementation.
in the civil engineering university curriculum. Taking into account this framework, it is then believed that the methodology, results, and conclusions obtained with respect to the failure assessment of the case study reported herein could offer an even more effective example toward a working knowledge of landmark structural failures.

**Description of the Structure**

From a structural point of view, the *Cappella dei Principi*, as it is today, is a composition of four principal structural elements (Fig. 2): (1) crypt, (2) chapel (the lateral masonry walls that sustain the tambour), (3) tambour, and (4) dome.

The crypt is the lowest element of the structural compound to get access to the chapel (the original entrance was on the adjoining Basilica of San Lorenzo). The plan layout is an octagon, obtained by the intersection of two square elements (Fig. 3). The main dimensions of the crypt are a diagonal length of about 28.8 m and a height of about 6.0 m. The masonry walls thickness is about 7.3 m. Four internal pillars (with square cross sections with a side of ~2.3 m) support the depressed masonry vault roof. Both the apses (one for each side of the octagon) and the two stairs that allow access to the upper chapel level (the funeral chapel of the Medici family) are obtained out of the thickness of the lateral walls. Because of the presence of the apses opening, the effective cross section of the crypt is composed of a sequence of strong and weak masonry walls. The strong walls (a kind of strong pillars), placed on the octagon vertex, continue on the upper level of the chapel; on the contrary, in the weak walls, some openings that are used as side apses are present at the upper level.

The second structural element characterizing the building is the chapel (Fig. 4), with a diagonal length of 29.7 m and a total height (floor level to tambour basement) of about 21.5 m. The masonry walls thickness (~6.7 m) is lower than the corresponding crypt walls.

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**Fig. 2.** External southern view of the *Cappella dei Principi* (the circle highlights the external access to a little empty space under the bell-shaped window)
and the internal pillars present on the lower crypt level do not continue on the chapel. As for the crypt, the chapel perimeter is composed of a succession of strong and weak elements. In particular, where at the crypt level a weak wall is present, at the chapel level, openings corresponding to the lateral apses are present (Fig. 2). These apses, which develop from the chapel up to the tambour level, are closed on top by a barrel vault with an overall thickness equal to that of the upper tambour. This barrel vault is made of nonconnected stone masonry blocks with dimensions varying between 60 and 80 cm. The internal surface of the chapel is entirely covered with marble, cast into some slabs suspended by iron hangers to the structure.

The tambour is the third element, between the chapel and the dome. It has an octagonal layout, like the lower chapel, and its characteristic dimensions are a wall thickness of about 4.3 m and a height equal to 11.8 m (Fig. 5). The dimension of each side is about 11.4 m. On each side of the octagonal tambour there is an opening (a bell-shaped window) surmounted by a big arch that acts as the impost for the upper dome. Under each bell-shaped window, a little empty space (accessible only from outside, see the circled area in Fig. 2) is present, spanning the entire thickness of the tambour, probably what remains of the original opening existing on the 1654 layout. Through these empty spaces, it is possible to reach and inspect the extrados of the barrel vault covering the underneath lateral apses. From a structural point of view, the bell-shaped windows, cutting the tambour along the vertical direction, seem to reduce considerably the tambour stiffness and the consequent ability of the latter to counteract the radial dome’s thrusts.

The dome is the last element of the Cappella dei Principi. It is an octagonal cloister dome whose main dimensions are an inner diagonal of about 29.7 m and an overall height of about 22.6 m.
Up to the first 6.45 m, the dome structure has a full section with uniform thickness of about 3.20 m, made of masonry brick laid on horizontal beds; between 6.45 and 10.30 m, the full section of the dome is composed of two layers; after 10.30 m up to the upper compression ring, the dome structure includes three different masonry layers having different centers. The first internal layer has a thickness of about 90 cm, while the second intermediate layer has a thickness of about 50 cm. The first and second layers date back to the 17th century; on the contrary, the last masonry layer, built by Ferdinando and Giuseppe Ruggieri, dates back to the 18th century. The function of this external layer, whose thickness is about 30 cm, is mainly to preserve the building from environmental loads. The three layers are made of masonry bricks, and the in situ survey verified that the two internal layers were built with a herring-bone disposition, while the external one was built with horizontal beds. Between each layer a cavity is present, with dimensions ranging from 1.2 (empty space between the first two layers) to 0.5 m (empty space between the last two). In the cavity between the internal and the intermediate layer, a flight of steps is present that can reach the oculus. The dome, having an octahedral shape, is composed of eight webs connected at the vertex; in correspondence to each vertex between the webs, the two internal layers are structurally connected by masonry joining elements (spurs). These spurs start from the full section and continue up to the oculus of the dome (in correspondence to the open compression ring). Overall, 32 spurs are present: eight of them are placed at the edge between different webs, and the others are placed three at each web (at one-fourth, two-fourths, and three-fourths of the web length). The vertex spurs are 1.15-m thick at the base and tapering to 0.5 m at the upper compression ring, while the web spurs are 55-cm thick at the base and 12 cm at the oculus. The eight vertex spurs are interconnected by means of five levels of masonry arches (at ~165-cm intervals) that run on a plane of the web to create a kind of annular ring. These spurs seem structurally connected with the internal layer by means of stones that are clamped into the masonry; on the contrary, the intermediate layer seems simply leaned.

Fig. 4. Chapel layout (level + 12.0 m)
on the extrados of the spurs. On the extrados of the internal layer, five levels of iron ties are present, with a rectangular cross section with a side dimension of approximately 5 cm.

The final dimensions of the Cappella dei Principi are as follows: inner diameter of about 29 m (with a masonry wall thickness ranging from 7.3 to 4.3 m) and a total height, with respect to the external ground, of 62 m. Table 1 compares the dimension of the dome of the Cappella with those of other masonry domes in Europe.

### Damage Survey

The first historical information concerning the presence of cracks in the dome structure was reported in 1690 (40 years after its construction and 50 years before the renovation works) by Alessandro Cecchini, which compared the fissure in the Brunelleschi’s Dome with those existing in the Dome of the Cappella dei Principi (Nelli 1753). In particular, Cecchini spoke about “crepature larghe il doppio più, che non sono quelle della Cupola del Duomo” (cracks whose width is about the double of existing cracks in the Brunelleschi’s Dome), thus suggesting that the width of these cracks was about 8–10 cm. Although a precise description of these cracks with their position and extension is not reported (Cecchini wrote only that subsequent works sealed these fissures), this information reveals that the dome-tambour system had been suffering some structural problems since the beginning.

As highlighted during the geometrical description, the section of the chapel is characterized by a succession of strong (the octagon vertexes, a kind of strong pillars) and weak walls (the side apse). This sequence of structural elements with varying stiffness, coupled with the presence of major openings on the tambour, has originated a quite regular system of cracks in the structural compound. The actual cracking pattern mainly affects the dome and the tambour up to the barrel vault over the side apses and, because of the symmetry of the structure, it is almost regular throughout the whole building. To describe this cracking pattern, the eight sides of the chapel are numbered from 1 to 8 following a clockwise notation (Fig. 6), and
thus, for instance, from a geographical point of view, Side 7 is the northern side and Side 3 is the southern one. From an architectural point of view, even sectors correspond to strong walls, while the lateral apses are present in the odd sector.

On the sectors on the chapel level where a side apse is present (odd sectors), there are cracks that develop from the upper third of the dome, cross the bell-window, and reach the keystone of the apse barrel vault. On the sectors where the apse is not present (i.e., on the strong chapel walls positioned on the even sectors), the cracks, still developing from the dome, reach the bell-shaped windows and do not continue (Figs. 7 and 8). These cracks are oriented along the typical meridian direction with the maximum opening at the windows. The in situ survey has verified that the subvertical cracks affecting the dome do not pass through the width of the internal masonry layer, because they are visible only at the intrados.

Exhaustive and complete cracks recognition in the chapel was quite difficult to perform because of the internal marble covering, which, hiding the masonry texture, makes an overall survey impossible. In particular, it was difficult to investigate the effective depth of the cracks in the tambour and in the barrel vault over the apse, because the area can be reached only outside by an access platform. On the external side, only partial failures are visible on the outer masonry arch of the apse side (minor local sliding of stones at one quarter span the position of the arch). The investigation, even if local, of both the masonry texture and the internal constructive technique, together with a survey of the cracks in the tambour region, was made after the local failure of 1999, when the dropping of a marble stone of the internal covering in a lateral apse (Sector 3 in Fig. 6) revealed the presence of additional static problems in the construction, because one of the keystones of the barrel vault over the apse supporting the tambour had fallen. Consequently, to investigate the monument and carry out the necessary restoration, all the covering marbles in this sector were completely removed, showing the big arches masonry texture and allowing for some experimental surveys. The covering marble elements are arranged

<table>
<thead>
<tr>
<th>Structure</th>
<th>Inner diameter (m)</th>
<th>Total height (m)</th>
<th>Construction (CE)</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pantheon (Croci et al. 1997)</td>
<td>43.4</td>
<td>43.4</td>
<td>~120</td>
<td>Roman concrete</td>
</tr>
<tr>
<td>San Pietro (Croci et al. 1997)</td>
<td>42.6</td>
<td>136.5</td>
<td>1546</td>
<td>Stone and brick masonry</td>
</tr>
<tr>
<td>Hagia Sophia (Croci 2006)</td>
<td>32.6</td>
<td>55.6</td>
<td>532</td>
<td>Stone and brick masonry</td>
</tr>
<tr>
<td>St. Paul (Croci et al. 1997)</td>
<td>30.7</td>
<td>67.3</td>
<td>1675</td>
<td>Stone</td>
</tr>
<tr>
<td>S. Maria del Fiore (Bartoli et al. 1996)</td>
<td>45.5</td>
<td>103.4</td>
<td>1420</td>
<td>Brick masonry</td>
</tr>
<tr>
<td>Cappella dei Principi (Florence, Italy)</td>
<td>29.7</td>
<td>62.2</td>
<td>1604</td>
<td>Stone and brick masonry</td>
</tr>
<tr>
<td>Rotunda of Galerius (Thessaloniki, Greece)</td>
<td>24.5</td>
<td>30.0</td>
<td>306</td>
<td>Brick masonry</td>
</tr>
</tbody>
</table>

Fig. 6. Chapel sector’s identification

Fig. 7. Internal crack (Sectors 1/4)

Fig. 8. Internal crack (Sectors 5/8)
into plane elements with a surface of about 1 m² with a thickness of about 5 cm; they are fixed on a base of stone about 15-cm thick anchored to the masonry by iron hangers. Figs. 9 and 10 show the sliding of the barrel vault keystone together with the provisional elements placed after the marbles dropped.

The barrel vault over the lateral apse is substantially a semi-circular arch (Fig. 11) made in sandstone with a span length equal to 13.10 m and a cross section of about 180 × 330 cm. The section of this barrel vault was built by parallelepipeds-shaped stone by means of three not-connected orders of stones both in width and length. The internal layer of the prismatic stones (Layer III in Fig. 11) has a thickness of about 80 cm; the remaining two layers both have a thickness of about 50 cm. The sandstone voussoirs, almost perfectly squared, are simply leaned one against the other without mortar. The construction details of the arch explain, at first, the sliding of the keystone: first of all the arch has been constructed like three separated arches, because no connection stone exists between the layers; secondly, the stone voussoirs that constitutes the layers are perfectly squared and not tapered, as usually is in cases of arches (especially for the keystone). Fig. 12 depicts a section of the barrel vault in correspondence to the keystone failure; the figure reconstructs the damage as detected during the experimental investigation and shows the sliding of the keystones affecting mainly the first two rows of the first two internal layers of stones.

The damage found at the arch crown on the stone barrel vault in Sector 3 was also present in the barrel vault in Sector 5 (the high altar area). It is thus reasonable to expect the same damage in the other odd sides of the octagonal chapel.

**Experimental In Situ Survey**

The experimental campaign on the Cappella dei Principi, aimed at assess the local masonry characteristics, included flat-jack tests and drilling of cores. The positions of both tests, their number, and distribution (that even if not fully destructive remains slightly destructive tests) have been selected to minimize obtrusiveness and potential loss of material in the monument (i.e., to preserve its integrity) and, at the same time, to obtain the effective structural information needed for the subsequent numerical modeling at various scales. In particular: (1) the location of the cored sample has been selected to obtain a stratigraphy of the multileaf tambour wall; (2) the location of the single flat-jack tests has been chosen to assess the stress state in the arches covering the apse where the failure of the keystone was observed (taking into account the operative conditions); (3) the location of the double flat-jack test takes advantage of previous single flat-jack tests (i.e., after the execution of the single flat-jack test a new flat-jack was inserted and the test was carried out) to reduce the obtrusiveness. Both single and double flat-jack and cored sample tests have been executed in positions selected in accordance with the Soprintendenza.

**Drilling of Cores**

Taking into account the difficulties related to the cultural value of the structure (and consequently the impossibility to plan an extensive experimental investigation), only two cored samples at the level of the tambour were extracted (Fig. 13). The first was taken from the apse in Sector 3, starting from the external surface of the building and crossing the masonry wall section. The second one was taken inside the masonry wall, taking advantage of the presence of the bell-windows. The cores were extracted using air core drilling instead of hydraulic rotary drilling in order to reduce the impact on the monument. In fact, even if hydraulic technology allowed for extraction of a more intact sample, it requires putting water inside the walls, which could damage the gypsum, used as masonry anchorage of the iron elements sustaining the marble slabs.
Even if the main limit of drilling is that it only gives a local stratigraphy of the wall interior (and therefore it is difficult to extend results of local inspections to the whole building), this inspection provided useful information on the constructive techniques. The first cored sample permitted identification of the presence of a multilayered wall (Fig. 14). The external face is made of stone with a thickness of about 33 cm; the internal face is again made of stone with a thickness of about 75 cm; the internal filling is composed of heterogeneous material (disordered masonry and stone tied by a good mortar). The second cored sample substantially confirmed the quality of the internal fill (Fig. 15).

Fig. 11. Geometric representation of the barrel vault over the apse (Sector 3)

Fig. 12. Section of the apse in Sector 3 in correspondence with the keystone of the barrel vault
**Flat-Jack Tests**

Tests with single and double flat-jacks were performed on the system of arches covering the apse in Sector 3 to estimate the stress values (single flat-jacks) in the masonry texture and to assess the elastic in situ modulus (double flat-jacks). The test is only slightly destructive, because after the test is completed, the flat-jack can easily be removed and the mortar layer restored to its original condition. Both the single and double flat-jack tests have been conducted with respect to the recommendations of the ASTM (1991a, b) and, for the execution of the in situ tests, the following equipment was used: (1) semioval flat-jack (dimensions: 350 × 260 × 3.5 mm); (2) hydraulic circular saw (ø350 mm and thickness 3.5 mm); (3) hydraulic hand pump [equipped with two manometers, with full scales of 25 and 100 bar (2.5 and 10 MPa)]; and (4) mechanical displacement transducers (with a base length of 200 and 400 mm, with resolution equal to 0.01 mm). The measure of pressure \( p \) applied by the flat-jack, taking into account the corrective factors, approximately corresponds to the local pressure in masonry, and therefore the average compressive stress in the masonry, \( \sigma_m \), can be evaluated as follows:

\[
\sigma_m = S_f = K_m \times K_a \times p
\]

where \( K_a \) = factor that accounts for the ratio between the bearing area of the jack in contact with the masonry and the bearing area of the slot (assumed equal to 1.00 in the present case); \( K_m \) = factor that accounts for the physical characteristic of the jack (equal to 0.96); and \( p \) = pressure required to restore the original distance between the gauge points.

During the investigation, three single flat-jack tests and one double flat-jack test were made. All the tests were performed on the system of arches where the failure of the keystone was observed.
The first investigated location was the keystone of the stone barrel vault (Position 1 in Fig. 16) covering the apse and supporting the tambour. The area over the barrel vault is accessible through a little empty space (accessible from outside through the little door circled in Fig. 2) spanning the entire thickness of the tambour. After the removal of the fill material (disordered material tied by poor mortar), it was possible to reach the extrados of the arch (Fig. 17). At this testing position, after the creation of the slot where the flat-jack had to be inserted, an opening of the masonry above and below the slot was observed, meaning that in this area a tensile stress was present. It must, however, be observed that the test execution had required the removal of the filling material, and this could have biased the results.

The second investigated position (Fig. 16) was the masonry arch immediately beneath the barrel vault previously tested. Taking into account the operative conditions, the test was carried out in a position intermediate between the keystone and the springs (Fig. 18), and the slot was created in the mortar joint between the voussoirs (Fig. 19). This arch was tested to understand if this arch was an effective structural element that contributed to the vertical load transfer from the tambour to the masonry walls of the chapel or if, as it seemed, its function was mainly to realize a geometrical fitting from the dome toward the apse. This behavior was suggested by the presence of other arches that shelf outward. The result offers a pressure of 2.3 bar when the original distance between the points above and below the slot were restored, corresponding to a stress of about 0.22
The low level of compression seems to confirm the hypothesis, suggesting that the main parts of the vertical loads are transferred from the tambour to the masonry walls of the chapel via the upper stone barrel vault.

The third and last investigated location was the area close to the spring of the barrel vault (Fig. 20). The test offers a pressure of 23–25 bar corresponding to a stress value of about 2.25–2.45 N/mm² of the compression in the stone voussoirs. Fig. 21 reports the outcome of the double flat-jack test where the vertical strains are plotted as functions of the stress in the wall (stress-strain curves). The Young’s modulus was found to be about 2,400 N/mm².

Numerical Analyses

Numerical modeling has been proven to be an effective tool to help the comprehension of structural behavior of ancient fabricas, and the inherent literature reports a plethora of discussions (Lourenço 2002) and illustrative case studies. Lourenço et al. (2007), through the discussion of the case study of the Monastery Jeronimos in Lisbon (Portugal), highlighted the role of advanced numerical simulations in historic structures for the understanding of the structural behavior of ancient buildings. The paper demonstrated that numerical models can also be used as a numerical laboratory, where the sensitivity of the results to input material parameters, boundary conditions, and actions can be efficiently analyzed, offering invaluable information in the conception and understanding of in situ testing and structural monitoring. The ability
of the finite-element technique to assess and interpret the structural behavior of historic constructions has also been shown recently by Ivorra et al. (2009), who discussed the seismic behavior of the San Nicolas Bell Tower in Valencia (Spain). The numerical model was first calibrated by means of dynamic tests performed directly on the real structure, and was next used to obtain the seismic response and its relationship to the seismic Spanish standard. The finite-element technique was used by Taliercio and Binda (2007) to analyze the Basilica of San Vitale in Ravenna (Italy), a Byzantine building, which suffers diffused cracking and excessive deformation. The authors, taking into account the results of in situ topographical and mechanical investigation, built a complete finite-element model of the basilica that has been conceived as a first step toward the understanding of the structural behavior of the fabrica. Chiorino et al. (2008) analyzed the Dome of the Sanctuary of Vicoforte (Italy), the largest elliptical dome ever built, combining limit analysis and finite-element technique. The dome has been analyzed through models, which are able to provide reliable interpretations of its behavior and damage state. The authors aimed at defining a model of the structure able to describe the actual behavior of the construction to predict its response to expected future loads (e.g., seismic actions) and to optimize future strengthening interventions. A careful use of numerical analyses dealing with practical engineering problems has been shown by Betti et al. (2010), who discussed the cracking pattern in a historic Italian palace. Through the use of the finite-element technique, the authors provided an interpretation of the manifested damage in the palace, and the results of the numerical analyses have been used to design an extensive in situ investigation on the building.

Based on this growing literature, in the present research, several numerical models of increasing complexity were developed to analyze the structural behavior and assess the causes of the keystone failure. First, some simplified elastic linear numerical models made of shell elements with SAP2000 (SAP2000) were analyzed. These models, aimed both at having a first appraisal of the load-transfer mechanisms and recognizing the role of the structural elements, were finalized to investigate the contribution of both the dome radial thrust and the tambour stiffness to the cracking phenomenon. At the same time, because of the uncertainties in the material parameters (because of the difficulties of gathering an extensive experimental survey on the whole monument, the investigation was limited to the arches in Sector 3), parametric analyses were performed, where the elastic moduli of the structural elements were changed within a reasonable range. Other unknown parameters were assumed by considering reasonable values for historic masonry. These preliminary elastic analyses, even if the hypothesis of linear behavior for masonry is to a large extent an unrealistic assumption, are exclusively aimed at describing the overall behavior of the entire structure, recognizing the role of its structural elements, and evaluating the entity of the principal stresses acting on its main elements from a qualitative point of view. Results of this simplified model were then compared with those obtained from more sophisticated numerical analyses performed using the commercial code ANSYS (ANSYS) and adopting solid hexahedral elements to model all the geometrical components. The final three-dimensional nonlinear solid model was developed based on an overall structural topographic survey encompassing the entire monument.

Taking into account the aim of the research, the modeling strategy used was the macromodeling technique, which is, moreover, convenient for large-scale models. According to this technique, the masonry units and the mortar elements are assumed to be smeared, and an isotropic (or anisotropic) material represents the smeared units and mortars in the masonry. Other strategies, mainly suitable for small-size models, rely on micromodeling approaches where units and mortar are modeled separately. A comprehensive recent discussion on these
aspects is reported in the literature (Adam et al. 2010; Lourenço and Pina-Hendriques 2006; Del Coz Díaz et al. 2011).

Next, the main results of both simplified linear and nonlinear analyses with specific reference to the structural configuration of Sector 3 (i.e., mainly focusing the attention on the keystone failure) are reported and discussed.

Linear Analyses
The linear models, developed using the commercial code SAP2000 (SAP2000), were built at the beginning of the research, and they were intentionally simple, because the overall structural topographic survey was not yet available. Plane elements (shell) were used, where the membrane and bending thickness were evaluated by properly assuming equivalent sections. It is noteworthy to highlight this step, because it required a particular care as a result of the shape and dimension of the structural elements that have been modeled by plane elements. The dome, for instance, is a three-layer masonry, and its modeling by plane elements required detailed attention to the inertial properties of the representative finite elements.

The linear models have been built step-by-step to assess the contribution (from a qualitative point of view, because they are made assuming linear material behavior) of each structural element in the global structural behavior. In particular, the models described below were developed.

Model 1
As a first step, only the tambour and dome were modeled (and the tambour was assumed with fixed restraints at the basement); the model was used to make parametric analyses aimed at assessing the effect of the stiffness ratio between the two structural elements on the radial thrust $S$ transmitted by the dome to the tambour. The error that can be made by assuming the tambour as infinitely rigid with respect to the dome, compared with the case where the dome and tambour stiffness are the same, is lower than 30%. This is shown in Fig. 22, where the dependence of the dome’s thrust $S$ (compared with the maximum value of the thrust, $S_{\text{max}}$, reached when the elastic modulus of the tambour, $E_T$, is much greater than that of the dome, $E_D$), is plotted versus the stiffness ratio between the dome and tambour elastic moduli ($E_T/E_D$).

This result suggests that no significant errors are expected because of wrong evaluations of the relative stiffness of the elements.

Model 2
In a second refinement step, the previously mentioned model was extended to include the upper part of the masonry walls of the chapel; in particular, the masonry walls were modeled starting from the level of the springs of the barrel vault, and therefore the remaining parts of the masonry walls (substantially the strong pillars in Fig. 4) were modeled as link elements. This modeling was aimed at analyzing the stresses on the barrel vault under the tambour and assessing the effects of the overlying filling material, as well as the spandrel wall stiffness. Furthermore, the model was used to analyze the vertical stress distribution between the lower level of the tambour and the upper one of the chapel. Taking into account the experimental result from the double flat-jack test, the elastic modulus of the material constituting the barrel vault (sandstone without interlayer mortar) was assumed equal to 2,400 N/mm$^2$; the elastic modulus of the infill material of the dome and upper tambour elements was estimated to be equal to 800 N/mm$^2$. The equivalent elastic modulus of the links that simulates the strong pillars was evaluated equal to 1,000 N/mm$^2$. These last two values have been estimated based on the authors experience in similar masonry textures. Fig. 23 reports the compressive vertical stresses on the model due to the self-weight of the structure. The local coordinate of the shell elements of the barrel vault was modified in order that the stresses $s_{11}$ are along the circumferential direction (Fig. 24, where $S$ denote the spring section, $M$ the quarter-span section, and $K$ the keystone section).

Fig. 24 shows that the model, even if it is a very simple one, exhibits tensile stress on the intrados of the keystone. Values of circumferential stresses in the barrel vault for some representative section are reported in Table 2. Despite the punctual value of the stresses, the model shows an area in the intrados of the arch that is subjected to tensile stress. This internal stress distribution on the barrel vault can justify the existing damage: the unloading of the barrel vault in correspondence to the keystone area produces an opening between the voussoirs that, combined with both the specific shape of the keystone (a parallelepiped one instead of a wedge) and
the absence of a mortar layer between the voussoirs, has originated the sliding downward of the keystone.

The stresses found in the barrel vault area are due to both the vertical loads (mainly self-weight of the elements) and the relative stiffness of the structural elements that constitute the building. It was considered significant to analyze the contribution of the dome’s thrust on the tensile stresses obtained by the numerical model; consequently, the model was modified as follows. The self-weight of both the dome and the tambour elements was set equal to zero, and a corresponding equivalent (vertical) load was applied directly to the lower level of the tambour, and therefore the stiffness ratio is conserved, but the thrust is not present. Fig. 25 shows the stress configuration obtained in the barrel vault (without the dome’s thrust).
Table 2 compares the punctual values of stresses in the representative sections of the barrel vault with and without the effects of the dome’s thrust. The variations of stresses are lower than 10%. This seems to mean (even if the linear models can offer mainly qualitative evaluation) that a potential retrofitting of the structure with a circular tie rod around the dome aimed at reducing its thrust would not produce any effective help for the barrel vault, because the tensile stresses arose on the keystone are mainly the result of the vertical loads and of the relative stiffness ratio between structural components of the fabrica.

The simple numerical model was then used to develop some parametric analyses assuming different stiffness ratios. Because

<table>
<thead>
<tr>
<th>Section</th>
<th>With dome thrust $\sigma_{intra}$ (N/mm$^2$)</th>
<th>Without dome thrust $\sigma_{intra}$ (N/mm$^2$)</th>
<th>$\sigma_{extra}$ (N/mm$^2$)</th>
<th>$\sigma_{extra}$ (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring (S)</td>
<td>-4.24</td>
<td>-1.41</td>
<td>-4.45</td>
<td>-1.56</td>
</tr>
<tr>
<td>Quarter span (M)</td>
<td>-1.08</td>
<td>-0.56</td>
<td>-1.16</td>
<td>-0.60</td>
</tr>
<tr>
<td>Keystone (K)</td>
<td>0.49</td>
<td>-0.21</td>
<td>0.41</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

![Fig. 25. Compressive circumferential stresses $s_{11}$ (kg/cm$^2$) on the barrel vault without the contribution of the dome’s thrust (S = spring section; M = quarter-span section; K = keystone section)](image)

Table 2. Circumferential Stresses on Barrel Vault (Intrados and Extrados)
A change in the value of the elastic moduli of structural elements (mainly by varying their relative ratio) does not produce significant changes in the barrel vault stresses behavior; and (3) the tensile stresses on the barrel vault are mainly due to the vertical load.

It is interesting, finally, to report the distribution of the vertical load on the nodes of the numerical model at the tambour basement level together with the ones on the extrados of the barrel vault (Fig. 27). Basically, it is possible to observe a migration of stresses from the dome to the ribs and then to the strong walls of the tambour. Because of this load migration (facilitated by the bell-windows opening on the tambour), the barrel vault, which is present over the apse, is characterized by an absence of normal stresses at the crown level (as resulting also by the in situ test where no compressive stresses were found). The result is illustrated in Fig. 28. A similar result was also found by analyzing a theoretical case, where the bell-shaped windows were supposed not to have been opened since the initial stage of construction; this suggests that the load migration was mainly originated by the high stiffness of the lateral strong walls.

Model 3

The last refinement of (linear) modeling included the walls of the chapel. Model 2 has thus been improved by including these walls (once more modeled with shell elements with appropriate bending and membrane equivalent thickness). The model was used mainly to repeat the parametric analyses made with the previous one by changing the elastic moduli of the structural elements according to the values reported in Table 3. With respect to Case 1, Fig. 29 reports the vertical stress on the whole model, and Fig. 30 reports a detail of the radial stresses in the barrel vault. This model confirms the results obtained with the previous one. It is possible to observe a reduction of the axial load at the springs of the barrel vault, because in the previous model, their movements were blocked and now allowed. A comparison of the thrust lines in the case of Models 2 and 3 is reported in Fig. 31.

Nonlinear Analyses

The linear models made by shell elements were used to evaluate the mechanical response of the structure with respect to variations in a number of selected mechanical parameters (relative stiffness, effect of the dome’s thrust, etc.). These models, even though they are built on the assumption of linear behavior, allowed to assess the main characteristics of the structural response under vertical loads (self-weight) with a reduced computational cost. Furthermore, despite the attention that must be paid to the definition of the equivalent sections of shell elements, they offered a synthetic visualization of the internal stresses and load redistribution among the structural elements that facilitate the interpretation of the physical behavior. In addition, taking into account that it was possible to investigate experimentally only a local area of the fabrica, the linear analyses developed represented a sort of mandatory step useful to offer

Fig. 26. Axial load, bending moment, and thrust line for Cases 1, 3, and 4

Fig. 27. Vertical loads $F_{22}$ at the tambour basement and over the barrel vault
a validation of the results obtained with the more sophisticated numerical model (made by means of solid elements) used to perform nonlinear analyses.

The nonlinear model was built with the commercial code ANSYS (ANSYS), and the masonry elements were modeled using eight-node isoparametric elements (Solid45) with a homogenized approach. By using solid elements, the geometry of the structure was reproduced as accurately as possible, paying attention to any variation in wall thickness, irregularities, and wall connections according to an overall structural topographic survey encompassing the entire monument. In particular, the major architectonical details were reproduced, because the building complexity asks for a global and detailed model capable of representing both the overall spatial configuration and the entire set of architectural elements with structural relevance. The final 3D model consisted of 107,689 joints, 85,771 3D Solid45 elements, corresponding to 220,332 degrees of freedom. Material properties of masonry walls (Young’s modulus $E$, Poisson’s coefficient $\nu$, self-weight $W$) have been differentiated by taking into account each different element present on the building (according to Case 1 in Table 3).

In the first step, a linear analysis was developed with the aim to make a comparison with the results obtained with the previous models. Fig. 32 compares the thrust lines obtained with the linear shell model (Model 3) with the corresponding one obtained with the solid model in the case of linear behavior; the solid model confirms the structural behavior assessed by the simplified modeling. The distribution of the principal stresses on the barrel vault confirms the presence of compressive stresses at the spring and tensile stresses at the keystone, thus justifying the failure of the barrel vault. Fig. 33 reports the vertical compressive stresses in the whole structure; the load migration of the vertical load toward the stiffest pillars of the chapel is confirmed.

Subsequently, the solid model was used for nonlinear analysis. In particular, because this research focused on the analysis of the keystone failure, the actual cracking pattern was reproduced inserting gap elements (Link10; ANSYS) where cracks were present on the buildings. These elements (that can be used to model trusses, sagging cables, links, springs, etc.) are one-dimensional nonlinear uniaxial tension-compression elements with three degrees of freedom at each node (translations in the nodal $x$-, $y$-, and $z$-directions). They support tension-only (cable) or compression-only (gap).
options; and in the current study, they were used as a gap element by assuming compression-only behavior. To take into account the tensile strength of good masonry, in the following part of this research, an initial pretension corresponding to 0.1 N/mm² was assumed. This approach was preferred over the smeared crack model, because the crack position was quite clear and the aim was to offer only a diagnosis of the keystone failure. To insert these nonlinear gap elements, the nodes under the bell-shaped windows were doubled and then reconnected by Link10 elements. The stiffness of these gap elements was properly estimated, taking into account the corresponding area of each joint (interested readers can refer to Betti et al.}

Fig. 30. Compressive radial stresses \( s_{11} \) (kg/cm²) on the barrel vault

Fig. 31. Axial load, bending moment, and thrust line for Models 2 and 3, Cases 1

Fig. 32. Axial load, bending moment, and thrust line; comparison of results between SAP2000 (Model 3) and ANSYS models
The structural compound was then analyzed again under vertical loads derived from the self-weight and roof loads.

The final nonlinear 3D numerical model with gap elements was able to reproduce the relevant results of the experimental survey. A quite good adherence between numerical and experimental results was achieved on the spring area of the stone arch (Fig. 34). The compressive stresses vary from 2.1 to 2.5 N/mm², with a mean value close to the experimental results, where a variation from 2.25 to 2.45 N/mm² was recorded. Fig. 35 reports the results of the nonlinear model in terms of vertical compressive stresses in the whole structure.

The interpretation of the structural behavior of the chapel by means of increasing complexity numerical models (from shell elements to solid elements; from linear to nonlinear models) has allowed for a deeper understanding and interpretation of the observed failure in the barrel vault keystone. The quite simple preliminary models, built by means of shell elements and based on a simplified geometry, allowed a first assessment of the load-transfer mechanisms between the structural elements of the monument. The refined numerical models built with solid elements have subsequently confirmed these results, offering a final confirmation of the proposed diagnosis for the observed failure. Nevertheless, despite the analogies found with respect to the failure assessment, the quality of information offered by the two typologies of modeling is substantially different. The nonlinear 3D models (that are computationally demanding and have necessitated an overall structural topographic survey of the entire monument) allow for a deeper insight into the static of the fabrica. Being these models geometrically refined they offer, for instance, effective information about local stresses and represent an invaluable instrument to check the effectiveness of retrofitting proposal. Moreover, the final nonlinear 3D numerical model, being able to reproduce the building behavior both qualitatively and quantitatively, once integrated with the results of the monitoring system, will allow assessment of the behavior of the structural compound under possible severe loads, such as seismic actions. With this, further improvements of the nonlinear model will be aimed at removing the fixed restraints assumption, in order to take into account the soil-structure interaction, by properly modeling the ground restraint by spring elements (Link8; ANSYS) whose stiffness will be evaluated based on geotechnical ground characteristics.

At this step of the research, the numerical models allowed to assess that one of the main causes of the keystone failure is connected to the difference of stiffness of the walls of the chapel. From a structural point of view, these walls are a succession of strong (even sectors in Fig. 6) and weak walls (odd sectors in Fig. 6) used as lateral apses. Above the chapel, these lateral apses are closed by a barrel vault with a complex system of arches realizing the basis for the tambour. This geometrical difference has probably determined a different stiffness that in turn has modified the normal stress state of this structural element, as observed with the numerical model. The tambour area above the strong walls is stiffer than the corresponding one over the weak walls, and this could be considered the origin of the switching of the stress state. This unloading has probably originated the sliding on the keystone (that was facilitated by the specific shape of this keystone: a parallelepiped instead of a wedge). As a matter of fact, a long-term retrofitting intervention
needs to take into account these phenomena, and it must cover the following two points. First, it is necessary to create cohesion between the stone elements of the arch; second, it is important to increase the global resistance of the arch taking into account that, probably, in the rest of its life, the described phenomena will be amplified and a similar damage will be found in the barrel vault over the remaining three apses.

Concluding Remarks

The paper presented the results of a research the aim of which was the analysis and the interpretation of a local failure in the Cappella dei Principi in Florence, Italy. First, the in situ survey was described; then, the numerical analyses developed to obtain the static assessment of the monument were illustrated. The linear and nonlinear FEM, identified taking into account the in situ measures, allowed to reproduce both qualitatively and quantitatively the behavior of the structure and the static problems in the area of the main arches covering the side apses. The probable causes leading to the observed static problems have been described, because it was possible to identify the most significant aspects on the structural behavior of the building. One of these aspects is the structural relevance of the succession of strong and weak walls. Their connection is provided by stone barrel vaults that cover the apses and create the basis for the above tambour. These vaults, conceived by the constructor to convey the loads from the tambour to the strong walls, tend on the contrary to weaken the structure (as is proved by the substantial absence of compression at their crown). The effects of differential settlements seem not relevant for the development of cracks on the tambour-dome system: as a matter of fact, the cracks have also been found by assuming a fixed-base model (i.e., not considering soil-structure interaction). Results reported herein offer a first identification of the building’s behavior. A more exhaustive interpretation of the overall building functioning (considering also exceptional loads, e.g., earthquake loads) will require an interaction between several modeling strategies (i.e., limit analysis), each providing additional understanding on the structural behavior.

As far as a retrofitting strategy is concerned, it should be considered that the dome’s thrust does not significantly change the stress behavior of the barrel vault, and therefore a dome retrofitting with circular tie rods could be almost ineffective for the barrel vault. On the contrary, a long-term retrofitting intervention needs to take into account the assessed structural behavior by creating cohesion between the stone elements of the barrel vault and by increasing its global resistance.

Fig. 34. Nonlinear model: vertical stresses at the arch spring (kg/cm²)
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